Effect of Aerodynamic Drag and Mass in Trajectory Motion

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Abstract

Trajectory motion in the air is mainly affected by the coefficient of the aerodynamic drag, gravitation, mass of the object to be thrown and also the degree of angle use in trajectory. Any change between these factors could result in the change of trajectory motion and displacement. These papers is focusing on comparing the effect of aerodynamic and mass of the object and see which has the most significant impact on the trajectory motion and the displacement, the other factor is ignored and assumed to be in the same condition even when the value is large.

1. Introduction

There are two things that affect the trajectory motion. It is aerodynamic drag and mass of the object. Each of these two factors contributes in the change of displacement of trajectory motion. The problem that is being analyzed is about what factors affect the most in the trajectory displacement. Below is the description of these two factors:

1.1 Aerodynamic Drag

The Aerodynamic drag is one of the key factors in changing the displacement of the trajectory motion. It is considered as the enemy of every object that travelling through air due to fact that it cause an air friction to the object that crossing the air [2]. To change the value of aerodynamic drag there are several things that can be set. The first is the drag force. The second factor is the frontal area of the object. The third factor is the speed of object that is travelling and the last is the density of the air, and it is related to the humidity and the temperature of the weather in the air [1]. To summarize it, the formula of aerodynamic drag is written as below [1]:

\[ F_d = \frac{C_d \rho A V^2}{2} \]  

(1)

- \( C_d \) = Coefficient Drag
- \( F_d \) = Aerodynamic drag force
- \( \rho \) = Density of Air
- \( A \) = Frontal Area of Object
- \( V \) = Velocity of the Object

In more detail, the component variable in the formula above can be break down to smaller pieces to identify any kind of thing that affect the coefficient drag.

- **Density of the air**

  It is shown in the formula above that the increase in density of the air can make the aerodynamic drag bigger. This is because the more level of compact in the substance of air then there will more air resistance and friction and it will result higher value of aerodynamic drag. The formula of density of air is listed as below [3]:

\[ \rho = \frac{p}{RT} \]  

(2)
\[ \rho = \text{Density of Air} \]
\[ P = \text{Total air Pressure} \]
\[ R = \text{Gas Constant} \]
\[ T = \text{Temperature} \]

In this formula the increase of total pressure is directly proportional to the increase of density. Total pressure \( P \) is related to the altitude. The higher the altitude then the air pressure will be smaller [4]. The other thing that effect density is gas constant symbol as \( R \). This value of gas constant is very from every location depends on the mixture of the compounds of it [5]. The last thing that effect density of air is the temperature. Increase in the temperature can make the density is smaller. The greater the altitude, the temperature will be smaller, and then to decrease temperature high altitude is need.

- **Frontal area of the object**
  Frontal area of object is related to circumference are of the object. The relation is that the greater the value of area of the object, then the coefficient of drag will be smaller.

- **Velocity of object**
  The relationship between velocity of object with density of air is that the greater the velocity than the aerodynamic drag will be bigger.

- **Coefficient drag**
  In most definition, coefficient of drag is identified as dimensionless quantity that is used to quantify the drag or resistance of an object in the air or hydro environment. The value of coefficient drag is varying between its shape and it is merely experimental. In aerodynamic shape optimization the most important uncertainties are the flow related ones, arising from the different range of operation conditions related to the Mach number and angle of attack, as well as Reynolds number for a viscous flow modeled by the Navier–Stokes equations [7]. The relation between coefficient of drag and aerodynamic drag is that the greater the coefficient of drag then the aerodynamic drag will be bigger. Below is the example figure describing the different coefficient of drag between different shapes [6]:

<table>
<thead>
<tr>
<th>Shape</th>
<th>Drag Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>sphere</td>
<td>0.47</td>
</tr>
<tr>
<td>half-sphere</td>
<td>0.42</td>
</tr>
<tr>
<td>cone</td>
<td>0.50</td>
</tr>
<tr>
<td>cube</td>
<td>1.05</td>
</tr>
<tr>
<td>angled cube</td>
<td>0.80</td>
</tr>
<tr>
<td>long cylinder</td>
<td>0.82</td>
</tr>
<tr>
<td>short cylinder</td>
<td>1.15</td>
</tr>
<tr>
<td>streamlined body</td>
<td>0.04</td>
</tr>
<tr>
<td>streamlined half-body</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Fig. 1.** Drag coefficient value in different shape. Taken from: http://www.wikiwand.com/en/Drag_coefficient

### 1.2 Mass of Object

The other variable that is use the change the trajectory motion is the mass of object. The formula that describes mass is:
\[ m = D \times V \]  

(3)

In the formula above, it is described that there are two things that affect mass. First is density of the object. Density is related to compactness in a substance and volume is about the total space that object occupy. This two factors have directly proportional effect on the mass. This is the end about introduction of two variables that will be use in this paper. Next chapter will cover about the mathematic in this research.

2. Mathematical Model

The first force from that is coming from trajectory motion is weight. The formula of weight is \( w = m \) g. The component of acceleration from this are:

\[ ax = 0 \text{ and } ay = -g \]  

(4)

The second force that affect is the aerodynamic drag. The value of this force is equal to proportional to the square of the projectile’s speed relative to the air:

\[ f = C_d \, v^2 \]  

(5)

\( v^2 \) in the equation above is consist of 2 component from the velocity from the x-axis and y-axis \( v^2 = v_x^2 + v_y^2 \) or it can also mean as \( v = \sqrt{v_x^2 + v_y^2} \). The newton second law gives.

\[ \Sigma F_x = -C_d \, v \, v_x = m \, ax \text{ and } \Sigma F_y = -m \, g -C_d v \, v_y = m \, ay \]  

(6)

Then the component acceleration that involving gravity and coefficient of drag are:

\[ ax = -C_d/m \, v \, v_x \text{ and } ay = -g - (C_d/m) \, v \, v_x \]  

(7)

3. Analysis

To simulate the effect of aerodynamic drag and mass in the displacement of trajectory. The mathematical model \( ax = -C_d/m \, v \, v_x \text{ and } ay = -g - (C_d/m) \, v \, v_x \) is transpose into matlab program with the detail and initial condition as below.

- Mass \( (m) \) = 10 kg
- \( C_d \) = range from 0.1 to 0.9 (increment 0.2)
- Gravity = 9.81 m/s\(^2\)
- Weight = Mass * Gravity
- \( yp \) = zeros(4,1)
- \( yp(1) \) = y(2)
- \( yp(2) \) = \((−C_d/m)*y(2)*(y(2)^2+y(4)^2)^(0.5))
- \( yp(3) \) = yp(4)
- \( yp(4) \) = \((−w/m)*y(2)*(y(2)^2+y(4)^2)^(0.5))

In the program above, the variable that being modified is the aerodynamic drag, symbol as \( C_d \). The other variable is remaining constant. The result from the above program is in the below:
Fig. 2. The effect of different coefficient drag variable between 0.1 until 0.9 (interval 0.2) when other variable is at constant value

The chart above describe the trajectory motion with range coefficient of drag between 0.1 to 0.9 and 0.2 in each incremental and the other factors assume as constant (mass= 10kg, gravity = 9.81, x (0) = 100 m/s, y(0) = 10 m/s). It shows that X and Y displacement of the trajectory motion is reduce if the drag coefficient is increase due to the air resistance is bigger and that air resistance opposes the vector of motion the object.

In the next simulation the variable that being modified is the mass of the object. The other variable is in a constant value.

- Mass (m) = range from 10 to 90 (increment by 20)
- Cd = 0.5
- Gravity = 9.81 m/s²
- Weight = Mass * Gravity
- yp = zeros(4,1)
- yp(1) = y(2)
- yp(2) = ((-Cd/m)*y(2)^2+y(4)^2)^(0.5))
- yp(3) = yp(4)
- yp(4) = ((-w/m)*y(2)^2+y(4)^2)^(0.5))

Fig. 3. The effect of different of mass variable between 10 to 90 kg (interval 20 kg) when other variable is in constant value

The chart above describe the trajectory motion with range of mass between 10 to 90 and 20 in each incremental and the other factors assume as constant (Cd= 0.5, gravity = 9.81, x(0) = 100 m/s, y(0) = 10 m/s). It shows that X and Y displacement of the trajectory motion is reduce as the mass is decrease.
greater the value of the mass has impact in increasing the value of weight. It is reflected in the equation (7).

Then the summary of the effect of coefficient drag is as below:

Table 1: Effect of coefficient drag vary from 0.1 to 0.9 to the displacement of trajectory

<table>
<thead>
<tr>
<th>Cd</th>
<th>0.1</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>X Displacement</td>
<td>96</td>
<td>52</td>
<td>38</td>
<td>30</td>
<td>25</td>
</tr>
<tr>
<td>Y Displacement</td>
<td>3.3</td>
<td>2.12</td>
<td>1.65</td>
<td>1.35</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The average effect in decreasing coefficient of drag is:

Cd = 0.7 compare to Cd = 0.9
X displacement is change from 25 to 30 then it is an increase by 20 %
Y displacement is change from 1.2 to 1.35 then it is an increase by 12.5 %

Cd = 0.5 compare to Cd = 0.7
X displacement is change from 30 to 38 then it is an increase by 26.6%
Y displacement is change from 1.35 to 1.65 then it is an increase by 22%

Cd = 0.3 compare to Cd = 0.5
X displacement is change from 38 to 52 then it is an increase by 36.8%
Y displacement is change from 1.65 to 2.12 then it is an increase by 28.4%

Cd = 0.1 compare to Cd = 0.3
X displacement is change from 52 to 96 then it is an increase by 84.6%
Y displacement is change from 2.12 to 3.3 then it is an increase by 55.6%

Average X displacement increase = 42%
Average Y displacement increase = 29.6%

The summary of the effect of changing mass is as below:

Table 2: Effect of mass vary from 10 to 90 kg to the displacement of trajectory

<table>
<thead>
<tr>
<th>Mass</th>
<th>10</th>
<th>30</th>
<th>50</th>
<th>70</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>X Displacement</td>
<td>38</td>
<td>75</td>
<td>97</td>
<td>112</td>
<td>122</td>
</tr>
<tr>
<td>Y Displacement</td>
<td>1.65</td>
<td>2.75</td>
<td>3.25</td>
<td>3.6</td>
<td>3.8</td>
</tr>
</tbody>
</table>

m = 30 compare to m = 10
X displacement is change from 38 to 75 then it is an increase by 97 %
Y displacement is change from 1.65 to 2.75 then it is an increase by 66.7 %

m = 50 compare to m = 30
X displacement is change from 75 to 97 then it is an increase by 29.3 %
Y displacement is change from 2.75 to 3.25 then it is an increase by 18.1 %

m = 70 compare to m = 50
X displacement is change from 97 to 112 then it is an increase by 15.4 %
Y displacement is change from 3.25 to 3.6 then it is an increase by 10.7 %

m = 90 compare to m = 70
X displacement is change from 112 to 122 then it is an increase by 8.9 %
Y displacement is change from 3.6 to 3.8 then it is an increase by 5.5 %
Average X displacement increase = 37.65%
Average Y displacement increase = 25.25%

The final comparison result between decreasing coefficient of drag and increasing mass is as below:

![Chart showing comparison]

**Fig. 4.** The effect of decreasing aerodynamic compare to increasing mass in the displacement of the trajectory

Average of X displacement increase = 42% : 37.65% = 1.11 : 1
Average of Y displacement increase = 29.6 : 25.25 = 1.17 : 1

4. Conclusion

Mass of the object, gravity, initial velocity and as well as aerodynamic drag is several things that affect the trajectory displacement. In this paper other variables except aerodynamic drag and mass of the object remain constant. This is done to compare the effectiveness of mass of the object and the aerodynamic drag to the trajectory displacement. In order to modify the value of aerodynamic drag there is several things that can be change; there are coefficient drag, density of the air, frontal area of the object and velocity of the object. To change the value of the mass the required variable to be change is the density of the object and the total volume of the object.

In this paper to measure the effects of mass and aerodynamic drag. The experiment is conducted by increasing the mass and decreasing the aerodynamic drag. Based on the result that has been presented in the table, decreasing the coefficient of drag is proved to be more effective than increasing mass. The X displacement is increased around 11% better than increasing mass and in Y displacement; the increase effect is around 17 % higher. Air friction is once again proved to be the main reason that trajectory motions in the air become obstructed.

5. Acknowledgements

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Reference