Characteristics of Slider Crank Mechanism Using Modeling Simulations

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Abstract

This document will analyze the different effects of the length and the angular velocity affecting the performance of the slider crank system. The performance of the slider crank system is simulated and shown using MATLAB with reference of the model represented by a mathematical formula. The result of the simulation is represented by several graphs, showing the relation of the length and angular velocity of the rotary motion of the slider crank mechanism and the angle generated by the translational motion of the slider.

Keywords: Slider Crank; Derivative; Pivot; Angular velocity, Linear Velocity; Angular Acceleration; Linear Acceleration

1. Introduction

1.1 Background

Slider crank systems are commonly used today by different automobile industries and also machines which involves both rotary and translational motion of the components. Known to be commonly applied in engine pistons, the components of the slider crank systems are responsible to attain different results of oscillatory motions, rotation and translation. The performance of the slider crank mechanism relates to the speed output of the rotary motion of the rotating component and also the translational movement of the slider. Thus altering the values of the components will critically affect the oscillatory motion of the slider crank mechanism. According to the study of Akbari, lowering the speeds of the slider crank mechanism can be proven to decrease rate of oscillation of the system [1] because by decreasing the speed, the resulting slider will move slower resulting in lower rates of oscillation. In a larger aspect of application, this factor is crucial in order to attain the needed speed of oscillatory motion of the slider which is relevant to the required system. Thus, this paper will investigate the performance of the slider crank mechanism, when subjected to different variance of the components, mainly the effects of different lengths of the crank pivot bar and the different inputs of angular velocity, by showing graphical representation of the simulations done in the MATLAB program representing the running system.

2. Theory

2.2 Components of a Slider Crank System

The slider crank system consists of 3 main components which are the Crank Pivot, Connecting Bar and the Slider. Similar to the research done by Murk as’ team, the slider crank can be viewed as a joint component, each component having individual distinct fields of motion [2], thus obeying the simple physical mechanics in approach of analyzing the different effects.

The crank pivot is responsible for the rotary motion of the mechanism [3]. Assuming that the rotary motion of the crank pivot always rotates on the positive axis of rotation, the connecting bar attached to the crank pivot will result in an opposite rotary direction with respect to the crank pivot. The resulting slider component will reciprocate the polar axis of the rotary crank pivot. Thus if the crank pivot rotates in the positive axis of rotation, the resulting slider will translate in the positive polar axis.
2.3 General Model of a Slider Crank System

A general slider cranks mechanism diagram shows as follows:

![General model of Slider Crank Mechanism](image1)

According to Fig 1, the blue section of the graph shows the components of the slider crank mechanism which is subjected to motion. The followings shows the representation of the physical components of the mechanism:

- O- Crank Pivot
- OC- Crank Pivot Bar
- CP- Connecting Bar
- P- Slider

ω Pertains to the angular velocity of the rotating pivot bar OC. As the bat OC rotates, the corresponding bar which is connected to OC; the connecting bar CP will move in a linear translation motion, and also generates an angle of the resulting translation motion of φ. The length of OC relates to the radius of the circular path in which the crank pivot bar moves. With the physical relation known

\[ \omega = \frac{v}{r} \]

\(\omega\) is the angular of the velocity of the rotational motion is dependent upon the radius of the circular path, in this case of Fig 1, the length of the bar OC. The resulting translational motion is that the length between the centre O and point P will fluctuate according to the axis rotation direction of the crank pivot bar OC. Note that the angles of \(\theta\) and \(\phi\) are all in the units of Radians.

Based on the analysis done by Harihara and Childs, the geometrical model of a slider crank system is shown as follows [4]:

![Mathematical model of Slider Crank Mechanism](image2)

Fig 2. Mathematical model of Slider Crank Mechanism.
2.4 Geometric relations and physical relations to the model:

The following shows the representation of the variables:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>Length of the Crank Pivot Bar</td>
</tr>
<tr>
<td>$S$</td>
<td>Length of the Connecting Bar</td>
</tr>
<tr>
<td>$\emptyset$</td>
<td>Angle generated by resultant of translational motion of slider</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle generated by resultant of rotary motion of the pivot.</td>
</tr>
</tbody>
</table>

Based on trigonometric laws, the general geometric relation can be defined as follows:

$$S \cos \emptyset = a - L_1 \cos \theta \quad (1)$$
$$S \sin \emptyset = L_1 \sin \theta \quad (2)$$

According to the team of Hroncova, the diagram shown in Fig 1 can be modeled mathematically through trigonometric principles [5]. Where $S$ represents the length of the connecting bar and $L_1$ is the length of the crank pivot. The equation above represents the displacement of the slider with respect to the datum where the head of the crank pivot connects with the tail of the connecting bar. The angular velocity of the rotating crank pivot will induce a linear velocity on the slider. Given the length of the connecting bar, the angular velocity can be deduced by taking the derivative of equation (1) and (2) with respect to time [6]. The results of the first derivative to obtain the equation of angular velocity are shown as follows:

$$S \cos \emptyset - \sin \emptyset = L_1 \omega \sin \theta \quad (3)$$
$$S \sin \emptyset + \cos \emptyset = L_1 \omega \cos \theta \quad (4)$$

Equations (3) and (4) are 2 simultaneous equations which represents the horizontal and vertical components of the motion. Thus, both equations can be written in matrix multiplication form for a much easier conception shown as follows:

$$\begin{bmatrix} \cos \emptyset & - \sin \emptyset & \quad S \\ \sin \emptyset & \cos \emptyset & \quad \emptyset \\ \sin \emptyset & \cos \emptyset & \quad S & 1 & \quad \cos \emptyset \end{bmatrix} = L_1 \omega \sin \theta \quad (5)$$

Equations (3) and (4) can further be differentiated to represent the Angular Acceleration of the system. Angular acceleration usually represents a forced response during the running of the mechanical system, thus fluctuates an input resulting in a rotational acceleration of the movement of the crank pivot and also the resulting linear acceleration of the slider. The result of the second – derivative of the initial length is shown as follows:

$$\begin{bmatrix} \cos \emptyset & \sin \emptyset & \quad S \\ \sin \emptyset & \cos \emptyset & \quad \emptyset & \quad S \\ \sin \emptyset & \cos \emptyset & \quad \emptyset & \quad S & \quad 1 & \quad \cos \emptyset \end{bmatrix} = L_1 \cos \theta \omega^2 + 2S \emptyset \sin \theta + S \cos \emptyset \emptyset$$
$$-L_1 \sin \theta \omega^2 - 2S \emptyset \cos \emptyset + S \sin \emptyset \emptyset \quad (6)$$
3. Numerical Analysis

In this section of the paper, the results of the derivation of calculations from equation (1) to (6) are all implemented in to MATLAB. This allows the user to investigate and simulate with aid of graphical representation.

As shown in equation (5) and (6), the different simultaneous equation shows the results of both linear and angular velocity in matrix form. For the example shown in Fig 1, several parameters will be used to investigate the different effects of the length L1 and the angular velocity of the crank pivot. The values which will be inputted are tallied by a table shown as follows:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Input Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega ) (rads/s)</td>
<td>3 5 10</td>
</tr>
<tr>
<td>( L_1 ) (m)</td>
<td>0.5 0.7 0.9</td>
</tr>
</tbody>
</table>

The equations shown in (5) and (6) obey the laws of matrices in solving simultaneous equation. To calculate for the unknown, the Inverse is applied. Defined, as a sample matrices multiplication is given by the characteristic equation:

\[
AX = B
\]

(7)

The general equation of the Inverse Matrices Multiplication is shown as follows:

\[
X = A^{-1}B
\]

(8)

As referenced in the theory section, the system will be subjected to many variances such as vibrations, friction, abrasion forces and also change in heat conductivity in the metal material which can affect the overall stability performance of the mechanism. In this analysis section, the factors above are all neglected to ensure simplicity in conception, and also to fully understand the different effects of the altered values.

![Graph of the performance of a general Slider Crank Mechanism.](image)

With respect to equation (1) and equation (2), the value of \( S \) can be deduced shown on the graph above in terms of \( \theta \). To obtain the value of \( S \), substitute the value of \( \theta \) which is given in radians to equation (1) and equation (2). These values are significant to determining the maximum angle of \( \theta \) and \( \theta \) to obtain the optimal performance for the given length of the crank pivot bar.
As shown in Fig 4 and Fig 5, the graphs show a similar shape of the system pertaining to the linear velocity. Since velocity is a vector quantity, the negative curvature of the graph shows the velocity of translation in the negative polar axis.

When the increasing value of length $L_1$, the translational motion on the slider increases rapidly over a range of angle which is generated by the crank pivot. This is represented by the graph when the different values have a higher value of linear velocity with respect to the common value of $\theta$. In physical terms, it shows that the length of the crank pivot affects the velocity of the slider.

However, when the constant angular velocity $\omega$ is altered, the graph shows no change in the values of the resultant linear velocity.

Thus, to increase the slider translational velocity, given a constant angular velocity does not change the resulting value with respect to a fixed length.
As for the angular velocity shown in Fig 6 and Fig 7, the rotational motion of the crank pivot decreases with respect to the increasing angle of rotation given by $\theta$ when the value of the length $L_1$ is increased. This result is caused by the change in length of the crank pivot; as the length increases, the moment and resulting work done required increases due to the higher amount of force generated by a higher induced angular velocity [7]. As it increases, the rotational motion will show a decay towards the datum point of the rotation which is represented by the transition from decay to exponential trend of the graph. The other half of the exponential trend shows the positive axis of rotation when the crank pivot is rotating in the system.

Similarly with the results given in the Linear Velocity section, the graph does not show change in the behavior of the resulting system when the constant angular velocity $\omega$ is changed. This shows that regardless of the constant angular velocity $\omega$, the results will not show change in the performance of the system, thus changing the length of the crank pivot will induce more work done towards the rotational motion of the crank pivot.
The graphs in Fig 8 and Fig 9 shows the different results of the linear acceleration of the slider when the values of length and constant angular velocity are altered. The result shows a decaying trend in both results of the altered values. This decaying trend represents the negative translational acceleration of the slider.

However, the results shown in Fig 8 represents where the increasing value of the constant angular velocity has a smaller graph span compared to the graphs given with a smaller constant angular velocity. This shows that there is higher change in variable speed which pertains to higher acceleration of the slider when the angular velocity is increased.

Both graph represented a smooth curvature throughout the running of the slider crank system. Based on the study of Chaudhary, when the crank is subjected to a high velocity, the system induce higher forces of abrasion and vector inertia forces which will cause high amounts of vibrations in the testing of the real physical environment[8].
Fig 10 and Fig 11 shows the results of the angular accelerations at the crank pivot. The result shows that when the Angular Velocity is increased, the resulting Angular acceleration will increase for a given angle. The change in length of the crank pivot shows that the rotational acceleration increases within the given angle of $\theta$.

In the physical terms of the system, the longer the crank pivot bar the higher the acceleration will be generated by the crank pivot. There is also less time taken to change the rotational acceleration from the positive acceleration axis to the negative acceleration axis. Thus, by increasing length, more optimum rotational motion can be attained.

The result can also be interpreted by changing the initial constant angular velocity $\omega$. As the value of the angular velocity increases, the time span needed to change from the positive rotating axis to the negative rotating axis decreases, thus optimizing the rotational motion.
4. Conclusion

In conclusion, the length of the crank pivot and the initial angular velocity has a significant effect upon the translational motion of the slider crank mechanism. The goal of investigating the different effects of length and also angular velocity was individually represented in the graphs shown. Increasing the Length of the crank pivot bar and increasing the angular velocity can result in a much more stable performance of the slider crank mechanism. In most of the graph, for one cycle of the running mechanism, the values are able to reach the original datum point value of the original input which are being used, being length or angular velocity, thus this shows no spikes or fluctuations on the performance of the running mechanism. Noted that the initial angular velocity is given at a constant value, the physical quantity of acceleration still exist due to the shift in angle of \( \varnothing \). This document can be further analyzed by altering the values of \( \varnothing \), showing the different effects of the change in angle of the slider.

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References